Astronomy & Astrophysics manuscript no. paper
(DOI: will be inserted by hand later)
5th February 2008

Induced two-photon decay of the 2s level and the rate of cosmological hydrogen recombination

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Received 5 August 2005/ Accepted 2 September 2005

Abstract Induced emission due to the presence of soft CMB photons slightly increases the two-photon decay rate of the 2s level of hydrogen defining the rate of cosmological recombination. This correspondingly changes the degree of ionization, the visibility function and the resulting primordial temperature anisotropies and polarization of the CMB on the percent level. These changes exceed the precision of the widely used Cmbfast and Camb codes by more than one order of magnitude and can be easily taken into account.

Key words. Cosmic Microwave Background: recombination, temperature anisotropies

1. Introduction

One of the key processes for the formation of the primordial temperature fluctuations of the cosmic microwave background (CMB) is the recombination of hydrogen, which at redshifts $z \sim 1100$ makes the Universe transparent for CMB photons. The time and duration of recombination directly influence the characteristics of the CMB anisotropies. Today, line of sight Boltzmann codes like Cmbfast (Seljak & Zaldarriaga 1996) and Camb (Lewis et al. 2000) are routinely used to calculate the power spectrum of the primordial temperature anisotropies with inclusion of many different physical processes in the early Universe. A precision of the solution to $\sim 0.1\%$ within the assumed models up to multipoles $l \sim 3000$ is reached. This level of precision is now becoming necessary with the advent of space missions like Wmap, Planck and ground-based experiments such as Act and Spt, which will allow us to measure the CMB temperature and polarization anisotropies with unprecedented accuracy and thereby open a possibility to determine the key parameters of the Universe with high precision.

Recently Dubrovich & Grachev (2005) have included the two-photon decays of high levels of neutral hydrogen and helium in their calculation of the recombination rates, in the case of hydrogen yielding corrections on the level of a few percent and a significant acceleration of helium recombination. Leung et al. (2004) have included the softening of the matter equation of state due to the transition from completely ionized to neutral matter and found that the CMB temperature and polarization power spectra are affected on the level of some percent at large multipoles. As they pointed out, these corrections exceed the level of cosmic variance at multipoles $l \gtrsim 1000$ and therefore should be taken into account in high accuracy analysis of the CMB data. In this paper we discuss an additional physical process that changes the ionization degree of hydrogen in the Universe at any given moment of recombination on the level of a few percent.

It is generally accepted that the rate of recombination is mainly controlled by the two-photon decay of the metastable 2s level of hydrogen (Zeldovich et al. 1968; Peebles 1968; Seager et al. 1999, 2000). Here we discuss the influence of the simulated two-photon emission (e.g. see Berestetskii et al. 1971, p. 229) due to the presence of the low frequency photons of the CMB blackbody radiation. Below we present a simple calculation for the change of the two-photon decay rate of hydrogen, A_{2s1s}, with redshift and shortly discuss the consequences for the visibility function and CMB temperature and E-mode polarization power spectrum.

2. Induced 2s → 1s two-photon transition of hydrogen during recombination

Based on the pioneering work of Göppert-Mayer (1931) the rate for the 2s → 1s two-photon transition of hydrogen, assuming no ambient photon field, has been calculated and discussed many times using different approaches (Breit & Teller 1940; Kipper 1950; Spitzer & Greenstein 1951; Goldman & Drake 1981; Goldman 1989). Recently Labzowsky et al. (2005) gave a value of $A_{2s1s} = 8.2206\,\mathrm{s}^{-1}$

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for the total transition rate, which to 0.1% agrees with a more simple calculation based on the method used by Spitzer & Greenstein (1951). In this paper we use this simplified approach and include induced effects in the calculation of the hydrogen two-photon decay rate within the context of cosmological hydrogen recombination. A similar calculation can be used to include the effects of induced two-photon decay for helium recombination.

The total transition rate with no ambient radiation field can be given by

$$A_{2s1s} = \frac{A_0}{2}\int_0^1 \phi(y)\,\mathrm{d}y\,, \tag{1}$$

with $A_0 = 9\alpha^6 cR/2^{10} \approx 4.3663\,\mathrm{s}^{-1}$, where α is the fine structure constant, c is the speed of light and R is the Rydberg constant for hydrogen. In equation (1) $\phi(y)\,\mathrm{d}y$ is proportional to the probability of emitting one photon at frequency $y = \nu/\nu_0$ in the range $\mathrm{d}y = \mathrm{d}\nu/\nu_0$, where ν_0 is the frequency of a Lyman-α photon, with energy $\sim 10.2\,\mathrm{eV}$, while the second photon is emitted at $y' = 1-y = [\nu_0-\nu]/\nu_0$. The factor of 1/2 is required since there are two photons and each pair is counted twice. The function $\phi(y)$ is defined in the paper of Spitzer & Greenstein (1951) (cf. Eq. 3), which nowadays can be easily calculated numerically.

In the presence of an ambient radiation field with occupation number $n(\nu)$ the total transition rate is given by the expression

$$A^{\mathrm{ind}}_{2s1s} = \frac{A_0}{2}\int_0^1 \phi(y)[1+n(\nu)][1+n(\nu_0-\nu)]\,\mathrm{d}y\,, \tag{2}$$

where the factors of $1+n$ account for the effect of Bose-bunching. In the context of recombination the radiation field is given by a blackbody spectrum for which the occupation number is $n(\nu) = 1/[e^{h\nu/k_{\mathrm{B}}T}-1]$, with the photon temperature $T = T_0(1+z)$, where $T_0 = 2.725 \pm 0.001\,\mathrm{K}$ (Fixsen & Mather 2002). The relation between $h\nu/k_{\mathrm{B}}T \equiv x$ and y is given by

$$x = \frac{43455}{1+z}\,y \overset{z=1100}{\overset{\downarrow}{\approx}} 40\,y\,. \tag{3}$$

Figure 1 shows the comparison of the integrands of (1) and (2). Due to induced effect the probability of emitting one soft photon ($y \sim 0$) and at the same time the other close to the Lyman-α frequency ($y \sim 1$) is enhanced. This enhancement due to the change of x with time depends on the redshift. At sufficiently small redshifts, induced effects become negligible.

One can examine the behavior of $\phi(y)$ for $y \ll 1$ more closely by using the analytic fit as given by Nussbaumer & Schmutz (1984). With this, one obtains

$$\phi(y) = C\left[w(1-4^{\gamma}\,w^{\gamma}) + \alpha\,w^{\beta+\gamma}\,4^{\gamma}\right]\,, \tag{4}$$

where $w = y[1-y]$, $C = 46.26$, $\alpha = 0.88$, $\beta = 1.53$ and $\gamma = 0.8$ (note that in Nussbaumer & Schmutz (1984) the normalization constant C was defined differently). With

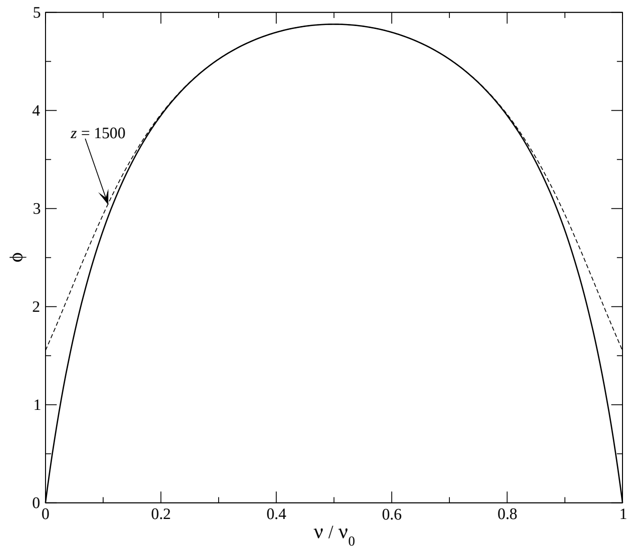

Figure 1. Two-photon 2s decay of hydrogen: the solid line shows the two-photon probability distribution, $\phi(y)$, as given in Spitzer & Greenstein (1951) assuming no ambient radiation field. In contrast to this, the dashed line includes the effects of induced emission due to the presence of the CMB at a redshift of $z = 1500$.

this the integrands of equations (1) and (2) around $y \sim 0$ can be expressed as

$$\phi(y) \approx C\,y\left[1 - 3.03\,y^{0.8}\right] \tag{5a}$$

$$\phi^{\mathrm{ind}}(y) \approx \frac{C}{\kappa}\left[1 - 3.03\,y^{0.8} + \frac{\kappa-2}{2}\,y\right]\,, \tag{5b}$$

respectively, where here we introduced

$$\kappa = \frac{43455}{1+z}\,. \tag{6}$$

These approximations are accurate within a few percent for $y \leq 0.05$ in the redshift range $z = 1000 - 1500$.

Here it is important that $\phi(y)$ vanishes as $\phi(y) \sim y$ for $y \to 0$. Since in this limit $n(\nu) \sim 1/y$, the product $\phi(y)n(\nu)$ is finite. Therefore it is not necessary to introduce a low frequency cut-off in the integral (2).

From (5) one can also deduce the contribution of the low frequency part to the total two-photon decay rate. Integrating from zero up to y_{m} one finds

$$\Delta A_{2s1s} \approx 50.5\,y_{\mathrm{m}}^2\left[1 - 2.16\,y_{\mathrm{m}}^{0.8}\right]\,\mathrm{s}^{-1} \tag{7a}$$

$$\Delta A^{\mathrm{ind}}_{2s1s} \approx \frac{101}{\kappa}\,y_{\mathrm{m}}\left[1 - 1.68\,y_{\mathrm{m}}^{0.8} + \frac{\kappa-2}{4}\,y_{\mathrm{m}}\right]\,\mathrm{s}^{-1}\,. \tag{7b}$$

Here Δ indicates that only part of the integrals (1) and (2), i.e. for $y \in [0, y_{\mathrm{m}}]$ have been taken. Due to the symmetry of $\phi(y)$ around $y = 1/2$ one can apply the approximations (5) and (7) also to the case $y \to 1$ by simply replacing $y \to 1-y$.

If one assumes that photons very close to the center of the Lyman-α line cannot escape then this reduces the contribution of the two-photon decay to the effective rate

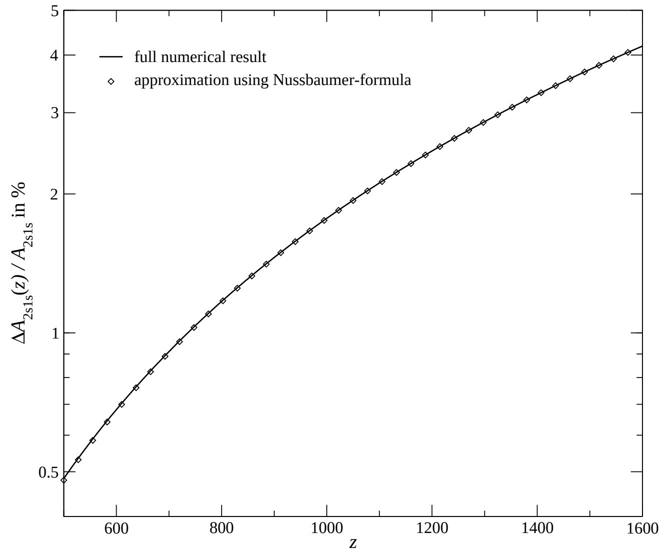

Figure 2. Redshift dependence of the relative change of the total two-photon transition rate of hydrogen, $[A_{2s1s}^{\rm ind} - A_{2s1s}]/A_{2s1s}$ with $A_{2s1s} = 8.2206\,{\rm s}^{-1}$, in the context of recombination using Eq. (2). In addition the results obtained with the simple analytic approximation (4) for $\phi(y)$ are shown.

of recombination, since the trapped photons prevent the corresponding recombination. As an example, if at redshift $z = 1100$ photons within 1% (5%) of the central frequency of the Lyman-α line cannot escape then the effective two-photon decay rate (i.e. the rate used in the calculation of the recombination history) is smaller by $\sim 0.06\%$ ($\sim 1.23\%$) for the standard calculation of A_{2s1s} and by $\sim 0.33\%$ ($\sim 2.01\%$) for the calculation including both spontaneous and induced effects.

As will be shown below, in general the contributions due to induced effects are at the level of some percent themselves. Hence, this estimate shows that the corrections to the two-photon decay rate we are discussing here can only be considered accurate if the more energetic photons from the 2s two-photon decay lying within less than $\sim 0.1 - 1\%$ of the Lyman-α line center are trapped. At redshift $z \sim 1100$ the Doppler width of the Lyman-α line is $\Delta\nu_{\rm D} \sim 2.3 \times 10^{-5}\,\nu_0$. Our computations of the Lyman-α photon escape in the distant wings show that there is no significant diffusion back to the line center beyond a few ten to hundred Doppler widths. Therefore the aforementioned condition should be easily fulfilled, since at $z = 1100$ a 1% distance from the line center corresponds to ~ 435 Doppler widths. However, even if every photon within 1% of the line center is unable to escape, this would only affect the results obtained below by about ten percent. One should note that the expected corrections due to the Lamb-shift are much smaller.

In Figure 2 we present the redshift dependence of the relative change of the total two-photon transition rate of hydrogen in the context of recombination using equation (2). A simple fit to this function, which within $\lesssim 1\%$ accuracy is applicable in the redshift range $500 \leq z \leq 2500$, can be given by the expression

$$\frac{A_{2s1s}^{\rm ind} - A_{2s1s}}{A_{2s1s}} = 1.181 \times 10^{-3}\,\chi + 2.177 \times 10^{-2}\,\chi^2 - 1.958 \times 10^{-3}\,\chi^3\,, \qquad (8)$$

with $\chi = (1+z)/1100$ and where A_{2s1s} and $A_{2s1s}^{\rm ind}$ are given by equations (1) and (2), respectively. The main correction scales as $\propto (1+z)^2$. The total correction exceeds the percent level for $z \gtrsim 700$. In addition the results obtained with the simple analytic approximation (4) for $\phi(y)$ are shown. They agree very well with the full numerical result but were obtained with much less numerical effort.

The rate for the inverse process can be found from equation (2) using the principle of detailed balance. In thermodynamic equilibrium the ratio of the population of the 1s and 2s levels is given by $N_{2s}/N_{1s} = \exp(-E_{21}/k_{\rm B}T)$, where $E_{21} = 10.2\,{\rm eV}$. Therefore the rate for the inverse process is given by $A_{1s2s} = A_{2s1s}^{\rm ind} \exp(-E_{21}/k_{\rm B}T)$. However, during the period of recombination, which is most relevant for the CMB anisotropies, the inverse process is negligibly small.

3. Changes of the ionization fraction, visibility function and power spectra

It is straightforward to include the redshift dependence of the 2s two-photon transition rate into the Cmbfast code using the approximation (8). One only has to replace the standard two-photon decay rate A_{2s1s} by $A_{2s1s}^{\rm ind}(z)$. With this one can calculate the changes in the ionization fraction, the visibility function, $\mathcal{V}(z) = \exp(-\tau)\,{\rm d}\tau/\,{\rm d}\eta$ (Sunyaev & Zeldovich 1970) and the temperature and E-mode polarization power spectra for the Wmap concordance model (Bennett et al. 2003). Above, τ is the Thomson optical depth and η is the conformal time.

In Figure 3 we have presented the redshift dependence of the ionization fraction and the corresponding change due to the inclusion of induced two-photon emission for the Wmap concordance model using a modified version of Cmbfast. The ionization fraction is affected by a few percent with a maximal relative difference of $\sim -1.3\%$ at $z \sim 1050$. Because the total two-photon decay rate is slightly higher than in the standard calculation, recombination occurs a bit faster.

In Figure 4 we show the results obtained for the visibility function. At redshifts much below the maximum at $z_{\rm dec} = 1089 \pm 1$ (Bennett et al. 2003) the visibility function is affected by less than one percent, whereas for $z \gg z_{\rm dec}$ the change reaches 8%. The position of the maximum and the width of the visibility function are both affected on a level below one percent.

In Figure 5 we give the results obtained for the temperature and E-mode polarization power spectra. Again the changes due to induced two-photon emission are on the level of a few percent, with an increase towards smaller scales. The amplitude of the change of the polarization

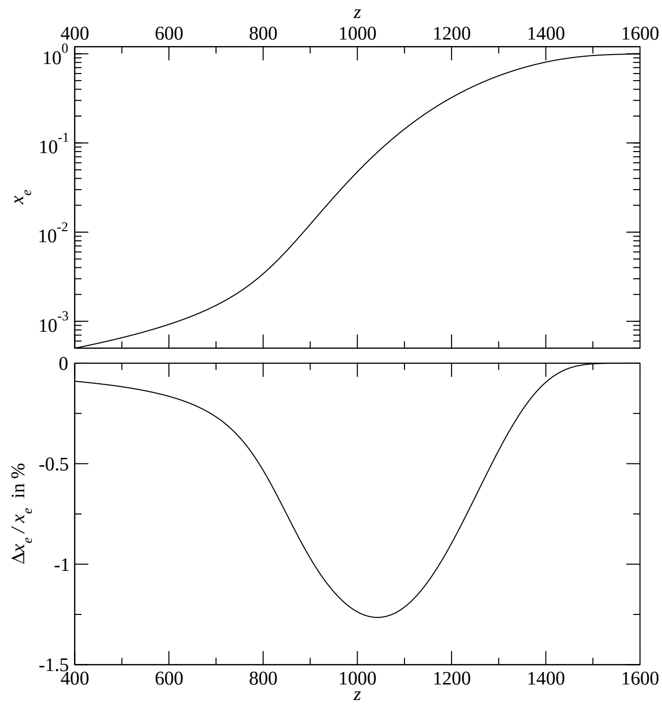

Figure 3. Ionization fraction, $x_{\rm e} = N_{\rm e}/N_{\rm H}$, and the relative change due to the inclusion of induced two-photon emission for the WMAP concordance model. Here $N_{\rm e}$ and $N_{\rm H}$ are the free electron and hydrogen number densities, respectively.

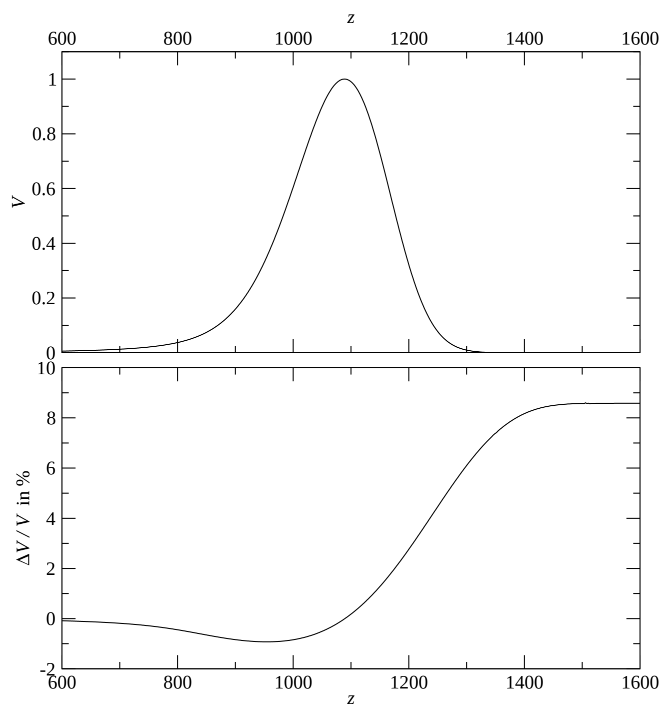

Figure 4. Visibility function, $\mathcal{V}(z) = \exp(-\tau)\,\mathrm{d}\tau/\,\mathrm{d}\eta$, and the relative change due to the inclusion of induced two-photon emission for the WMAP concordance model. The amplitude of $\mathcal{V}(z)$ is normalized, such that the maximum is 1.

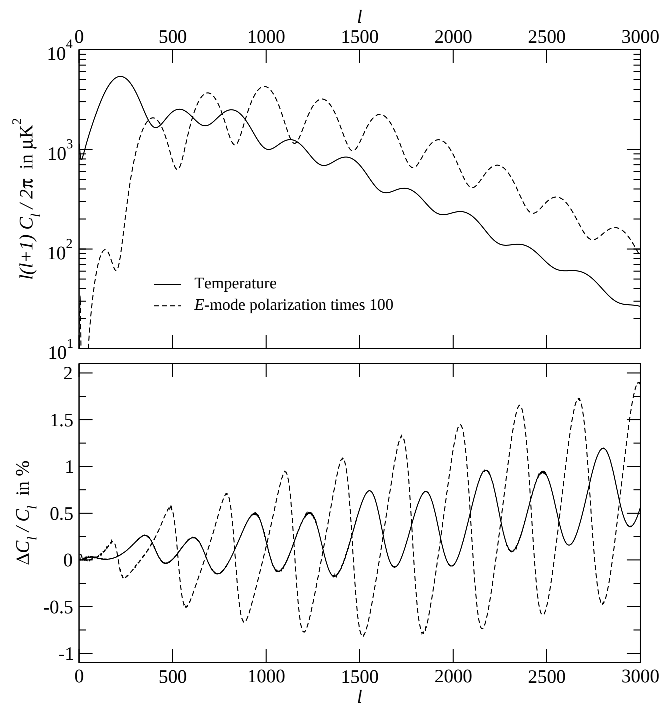

Figure 5. Temperature and E-mode polarization power spectra and their relative change due to the inclusion of induced two-photon emission for the WMAP concordance model.

power spectrum is roughly twice that of the temperature power spectrum.

4. Conclusion

Due to induced two-photon decay of the hydrogen 2s level, the rate of recombination is increased on the level of a few percent around the maximum of the visibility function. This increase results in changes of the ionization fraction, the visibility function and the temperature and polarization power spectra by a few percent. These changes can be easily taken into account for future high accuracy analysis of CMB data using the approximation (8). Induced two-photon decay would similarly influence the recombination of HeII and HeIII, but in that case the effects on the CMB power spectra are expected to be extremely small.

Acknowledgements. The authors wish to thank the referee, Prof. Dubrovich, for his comments on the manuscript. We acknowledge the use of CMBFAST for the calculation of the power spectra.

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